Minicourse #5: Visualizing projective geometry through photographs and perspective drawings

Part A, Wednesday, 9:00 a.m.-11:00 a.m.; Part B, Friday, 9:00 a.m.-11:00 a.m.
Room 29C, Mezzanine Level, SDCC
Thanks to NSF!

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Viewpoints: Mathematical Perspective and Fractal Geometry in Art

by Marc Frantz and Annalisa Crannell
Princeton University Press, 2011
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The viewpoint is all-important...
Two or more lines in the real world that are parallel to each other but not parallel to the picture plane have images that converge to a common point called a vanishing point.
Two or more lines in the real world that are parallel to each other and also parallel to the picture plane (the window) have parallel images.

(insert image of 1 pt perspective)

Or can we say, “have images that converge to a common point, the point at infinity”??
What is the difference between mathematical perspective and projective geometry?
Mathematical perspective is...
the study of the mathematics behind representing 3D objects and space onto a 2D surface by means of projection from a single point.

Figure: Brook Taylor, 1719
Projective geometry is... the study of geometric properties that remain invariant under projections.
Gérard Desagures introduced many fundamental concepts of projective geometry in *Brouillon Project*, 1639, including points at infinity.
Figure: illustration from Deargues’ *La Perspective*, 1636
Desargues’ Theorem

Appeared in *Maniéré universelle de Mr Desargues, pour pratiquer la Perspective...* by Abraham Bosse, 1648
However, his ideas were largely ignored for nearly two centuries because

- His “style and nomenclature are weird beyond imagining” - Julian Lowell Coolidge
- René Descartes introduced his ideas on analytic geometry in *La Geométrié* in 1637.
Jean Victor Poncelet revived projective geometry while a prisoner of war in Russia, 1812-14.

- He developed his ideas not knowing Desargues
- He published his work in *Traité des Propriétés Projectives des Figures* in 1822
Two important ideas discussed in *Traité* are the Principle of Duality and the Principle of Continuity:

**Principle of Duality:** “point” and “line” can be interchanged.

Axiom: Two points determine a line.

Dual axiom: Two lines determine a point.
Principle of Continuity: certain geometric relationships do not change through “continuous, but otherwise arbitrary movements"
“perspective had no influence on the development that ultimately led to the creation of projective geometry. ... Even though central projections surely played an important role in Poncelet’s work and he clearly saw their connection to perspective, and also applied the term perspective broadly, this does not imply that the existing theory of perspective is what inspired him.”

Goal of this minicourse:
To explore the connections between perspective and projective geometry.

Topics:
- Perspective drawing
- Desargues’ Theorem
- cross ratio and harmonic ratio
- Casey’s theorem and angle
- Eves’ theorem