Lab Project: Shifting and Scaling

INTRODUCTION: The purpose of this lab is to graphically explore shifting and scaling functions and to gain some experience using Mathematica.

1. Suppose $f(x) = x^2 - 2x$ and define $g(x) = f(x + b)$ and $h(x) = f(x) + b$, where $b$ represents a constant.

   (a) Using $b = 3$, graph $f$, $g$, and $h$ on the same set of axes. How do the graphs of $g$ and $h$ relate to the graph of $f$?

   (b) Using $b = -3$, graph $f$, $g$, and $h$ on the same set of axes. How do the graphs of $g$ and $h$ relate to the graph of $f$?

   (c) Experiment with other values of $b$. Then summarize the relationships between the graph of $y = f(x)$ and the graphs of $y = f(x + b)$ and $y = f(x) + b$, for $b > 0$ and for $b < 0$. 
2. Suppose \( f(x) = x^2 - 2x \). Let \( p(x) = b \cdot f(x) \) and \( q(x) = f(b \cdot x) \).

(a) Using \( b = 2 \), graph \( f \), \( p \), and \( q \) on the same set of axes. How do the graphs of \( p \) and \( q \) relate to the graph of \( f \)?

(b) Using \( b = 1/2 \), graph \( f \), \( p \), and \( q \) on the same set of axes. How do the graphs of \( p \) and \( q \) relate to the graph of \( f \)?

(c) Experiment with other values of \( b \). Then summarize the relationships between the graph of \( y = f(x) \) and the graphs of \( y = f(bx) \) and \( y = bf(x) \), for \( b > 1 \) and for \( 1 > b > 0 \).
3. Test your summaries given in problems 1(c) and 2(c) above as follows: Graph \( y = \sin(x) \) and graph the following four functions:

\[
p(x) = \sin(2x) \quad q(x) = 2\sin(x) \quad r(x) = \sin(x) + 2 \quad s(x) = \sin(x + 2)
\]

Provide a rough sketch of each of these functions, noting the period, amplitude, and intercepts.

In each case, explain the effect the number 2 had in modifying the graph of \( y = \sin(x) \). Are the relationships here consistent with your summaries above? If not, then modify your summaries.
4. We will now study the relationships between the graph of \( y = f(x) \) and the graphs of \( y = -f(x), y = f(-x), y = |f(x)|, \) and \( y = f(|x|) \).

Let \( f(x) = \frac{2x}{3x + 1} \).

(a) Provide a graph of \( f \) and identify its axis intercepts and its horizontal and vertical asymptotes. On this same set of axes, provide sketches of the graphs of \( y = -f(x) \) \( y = f(-x) \) and describe the relationships between these graphs and the graph of \( f \).

(b) Provide another graph of \( f \). On this same set of axes, provide sketches of the graphs of \( y = |f(x)| \) \( y = f(|x|) \) and describe the relationships between these graphs and the graph of \( f \).
5. In Figure 1, the graph of \( y = h(x) \) is given.
(a) On this figure, sketch the graphs of \( y = -h(x) \) and \( y = h(-x) \).

(b) In Figure 2, the graph of \( y = h(x) \) is given. On this figure, sketch the graphs of \( y = |h(x)| \) and \( y = h(|x|) \).