Lab Project: Area Navigation

Introduction. Suppose that \(v(t)\) is the eastward velocity (in feet per second) at time \(t\) (in seconds) of an object moving along an east-west axis. The graph of \(v\) is given below in Figure 1.

1. Use the graph of \(v\) and simple geometry to solve the following problems:
   (a) Find the signed area of the region bounded by the graph of \(v\) and the \(t\)-axis from \(t = 0\) to \(t = 10\). Then find the net distance and the total distance traveled by the object over the interval \([0, 10]\).

   (b) Find the average velocity of the object over \([0, 10]\). (i.e. Find the average value of \(v\) over \([0, 10]\).)

   (c) Find the average acceleration of the object over \([0, 10]\). (i.e. Find the average rate of change of \(v\) over \([0, 10]\).)

   (d) Find the average speed of the object over \([0, 10]\).
2. Find a piecewise-defined algebraic formula for the velocity function \( v \) as represented in Figure 1.

3. Let \( p(x) = \) “the position (in feet) of the object at time \( x \) (in seconds) relative to the origin,” for \( 0 \leq x \leq 10 \), and assume \( p(0) = 3 \).
   (a) Express \( p(x) - p(0) \) as a definite integral involving \( v \).

   (b) Use the results from problems #2 and #3 (a) to find a piecewise-defined algebraic formula for the position function \( p \).

   (c) Find the average rate of change of \( p \) over the interval \([0,10]\). How does this compare with the average velocity found in problem #1 (b)?
4. Provide a graph of $p$ below. Identify the critical numbers of $p$ and the global maximum and minimum values of $p$.

5. Write out a set of “step-by-step” instructions that can be used to match the motion of the object with velocity $v$ as represented in Figure 1.