Fourier Series Notes

f[x] defined on [-L, L] has Fourier series

$$\frac{a[0]}{2} + \sum_{n=1}^{\infty}\left(a[n]\, \text{Cos}\left[\frac{n \pi x}{L}\right] + b[n]\, \text{Sin}\left[\frac{n \pi x}{L}\right]\right)$$

where

$$a[0] = \frac{1}{L}\int_{-L}^{L} f[x]\, dx$$

$$a[n] = \frac{1}{L}\int_{-L}^{L} f[x]\, \text{Cos}\left[\frac{n \pi x}{L}\right] dx\ ,\ n = 1,\ 2,\ \ldots$$

$$b[n] = \frac{1}{L}\int_{-L}^{L} f[x]\, \text{Sin}\left[\frac{n \pi x}{L}\right] dx\ ,\ n = 1,\ 2,\ \ldots$$

If $f[x]$ is even, $b[n] = 0\ \forall\ n$ and $a[n]$ can be calculated as the Fourier cosine series below.

If $f[x]$ is odd,
$a[n] = 0\ \forall\ n$ and $b[n]$ can be calculated as the Fourier sine series below.

There are many sufficient conditions for Fourier series to converge. Here are some: f and f' are piecewise continuous; f is pieceise smooth; f is piecewise continuous with finite one-sided limits at the endpoints of each subinterval; f is pieceise continuous with each piece monotone.

If f is continous at c, the Fourier series for x=c converges to f(c).
If f is not continous at c, the Fourier series for x=c converges to $\frac{f(c-)+f(c+)}{2}$.

f[x] defined on [0, L] has Fourier cosine series

$$\frac{a[0]}{2} + \sum_{n=1}^{\infty} a[n]\, \text{Cos}\left[\frac{n \pi x}{L}\right]$$

where $a[n] = \frac{2}{L}\int_{-L}^{L} f[x]\, \text{Cos}\left[\frac{n \pi x}{L}\right] dx\ ,\ n = 0,\ 1,\ 2,\ \ldots$

f[x] defined on [0, L] has Fourier sine series

$$\sum_{n=1}^{\infty} b[n]\, \text{Sin}\left[\frac{n \pi x}{L}\right]$$

where $b[n] = \frac{2}{L}\int_{-L}^{L} f[x]\, \text{Sin}\left[\frac{n \pi x}{L}\right] dx\ ,\ n = 1,\ 2,\ \ldots$