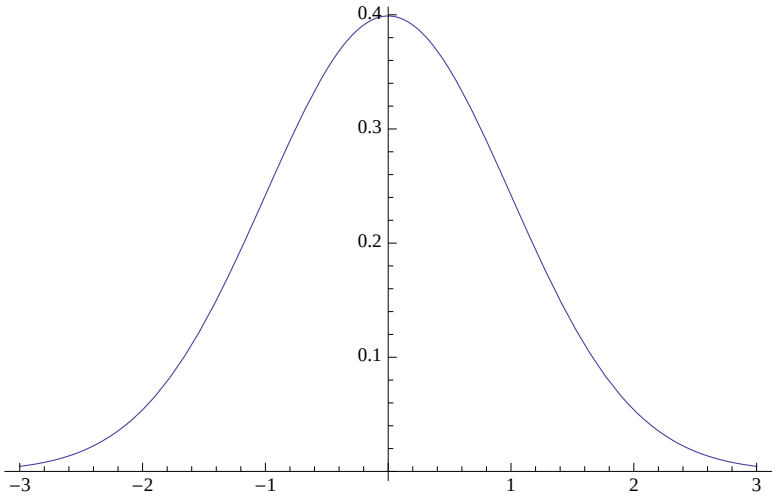
■ example 1: sampling from a normal distribution

```
Print["The parent population is the normal distribution. "]
Plot[PDF[NormalDistribution[0, 1], x], {x, -3, 3}]
```

```
The parent population is the normal distribution.
```



```
Clear[SampleSets, SampleMeans, SampleStDevs,
 SampleMaxs, SampleMedians, nSize, i, numSets]
nSize = 10;
numSets = 200;
SampleMeans = {};
SampleStDevs = {};
SampleMaxs = {};
SampleMedians = {};
For[i = 1, i ≤ numSets, i++,
 SampleSets[i] = RandomReal[NormalDistribution[0, 1], nSize];
 AppendTo[SampleMeans, Mean[SampleSets[i]] ];
 AppendTo[SampleStDevs, StandardDeviation[SampleSets[i]] ];
 AppendTo[SampleMaxs, Max[SampleSets[i]] ];
 AppendTo[SampleMedians, Median[SampleSets[i]] ]
]
Print["One of the samples of values from this distribution,
   along with its mean, standard deviation, max, and median"]
SampleSets[3]
SampleMeans[[3]]
SampleStDevs[[3]]
SampleMaxs[[3]]
SampleMedians[[3]]
```

```
One of the samples of values from this distribution,
  along with its mean, standard deviation, max, and median
```

```
{0.0783966, -0.0519998, 0.00738217, -1.60553,
 -0.536173, 0.13085, 0.538035, -0.653605, -1.12759, -1.73076}
```

```
-0.4951
```

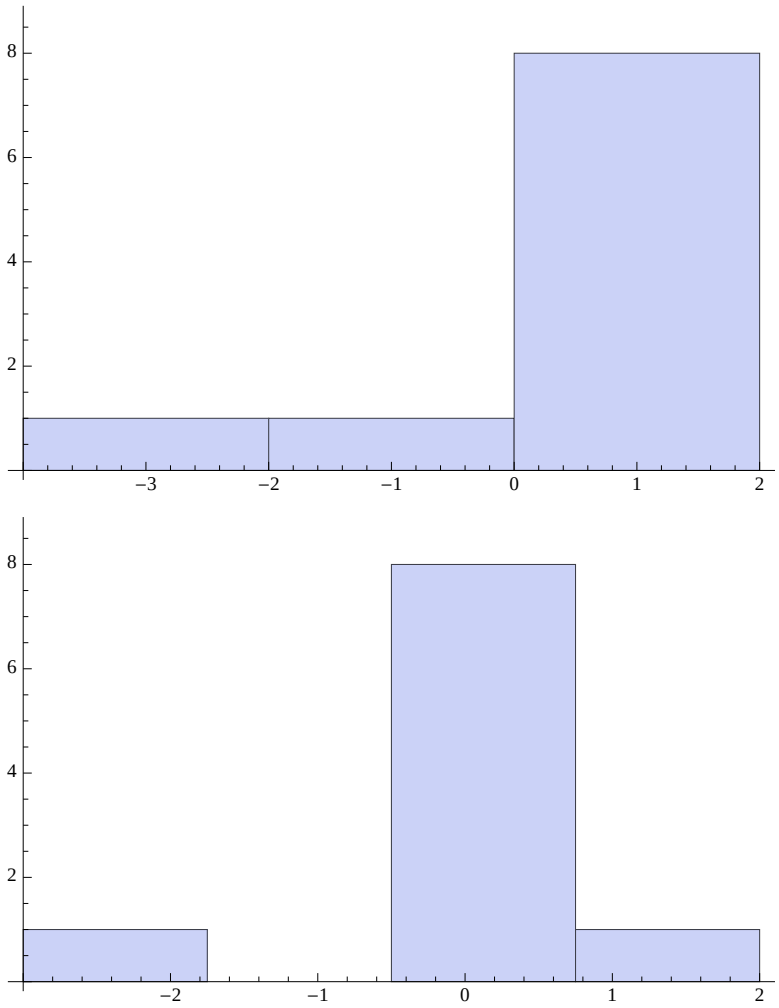
```
0.776293
```

```
0.538035
```

```
-0.294087
```

```
Print["Two histograms of one of the samples, shown with absolute frequencies."]
Histogram[SampleSets[3]]
Histogram[SampleSets[3], 4]
Print["A sample should resemble its parent population."]
```

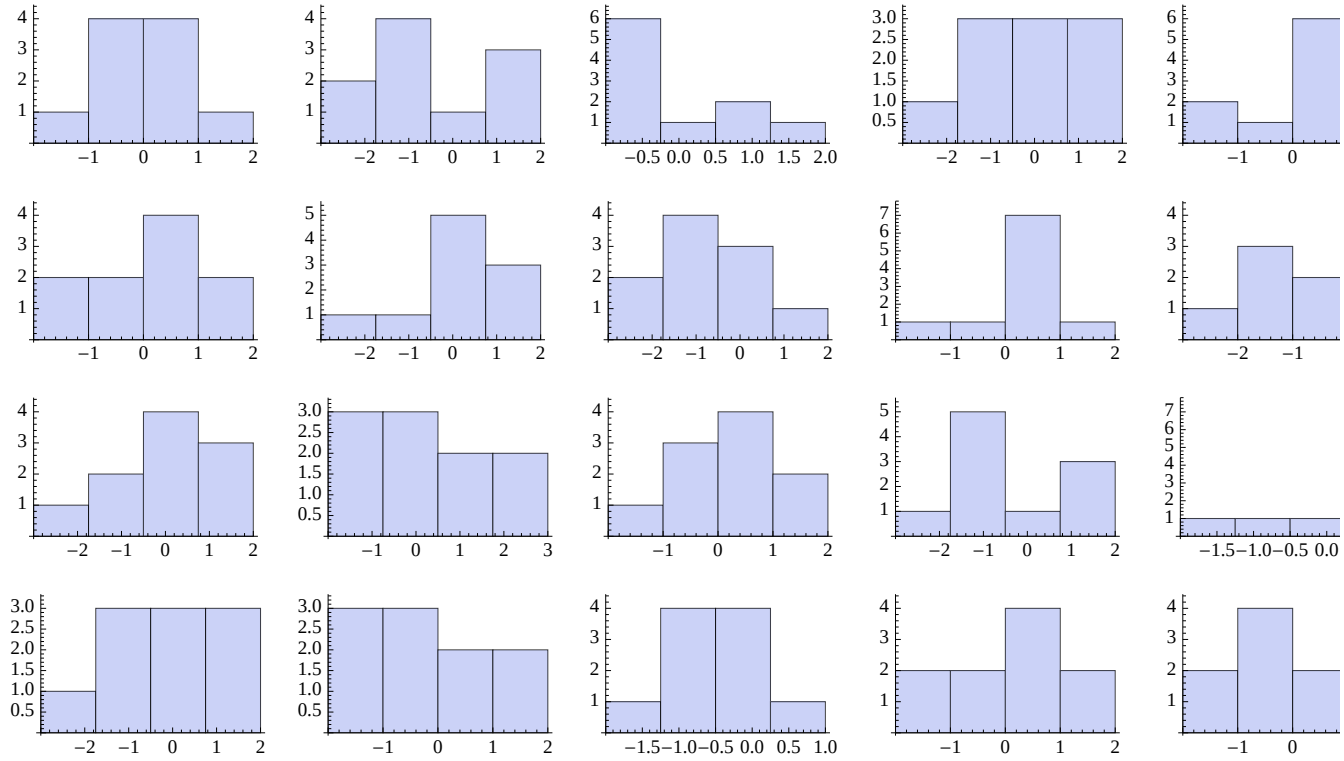
Two histograms of one of the samples, shown with absolute frequencies.



A sample should resemble its parent population.

```
Print["Samples vary, especially for a small sample size!"]
For[i = 1, i ≤ 4, i++,
 Print[GraphicsArray[Table[Histogram[SampleSets[5 i + j], 4], {j, 1, 5}]]
 ]]
```

Samples vary, especially for a small sample size!



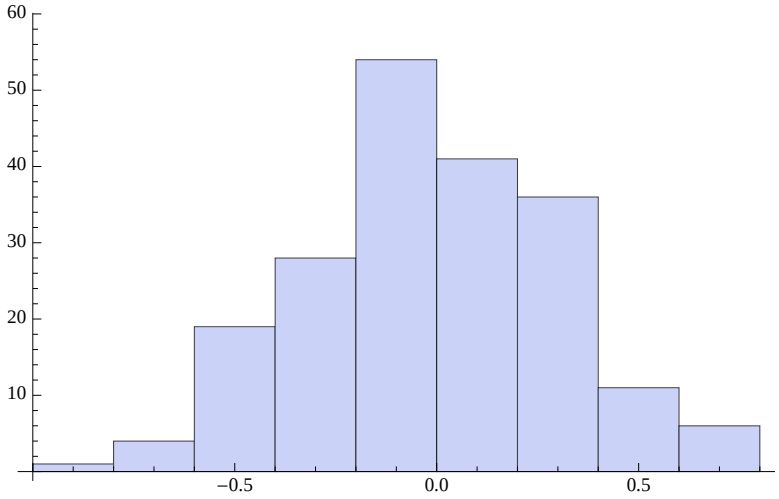
- **sample means - Central Limit Theorem applies**

SampleMeans

```
{0.113111, 0.581411, 0.0339161, 0.169981, -0.361071, 0.198558, -0.115854, 0.117825,
 -0.167405, 0.277937, 0.593186, 0.52158, -0.0782857, -0.0415499, 0.232084,
 0.178484, 0.24207, 0.295085, -0.338635, -0.176463, -0.340173, 0.295648, -0.431745,
 0.284404, 0.0634183, 0.0128039, 0.108879, 0.248397, -0.373137, -0.0117872,
 -0.191084, -0.0213241, 0.300645, -0.215727, -0.150794, 0.00061295, 0.0898844,
 0.385335, -0.358284, -0.117015, -0.0201292, 0.463285, 0.248259, -0.14314,
 -0.0242961, 0.0251449, 0.123631, 0.24639, -0.109037, 0.0864784, 0.344208, 0.154847,
 0.364381, -0.491785, -0.435499, 0.432447, 0.0587619, 0.487807, -0.0157112,
 -0.140663, -0.390392, -0.178449, -0.316315, 0.204075, -0.0537684, 0.0959024,
 0.565854, -0.0602696, 0.0590398, 0.156348, -0.231226, -0.584024, -0.058998,
 -0.36577, -0.0476452, 0.508452, 0.212144, 0.385205, 0.0661842, 0.535894,
 -0.0465926, 0.149108, 0.208432, -0.356347, -0.614496, 0.115082, -0.47044,
 -0.105534, -0.242119, 0.391683, 0.591986, -0.318303, -0.0310681, -0.0530339,
 -0.182339, -0.299641, 0.0388404, -0.565475, -0.591005, 0.117027, -0.558466,
 0.151025, 0.145297, -0.286338, 0.0739104, 0.0391033, 0.382462, -0.0195317,
 -0.40725, 0.0363026, -0.128661, -0.121417, -0.150403, 0.141799, 0.42392,
 -0.161277, -0.738232, 0.592323, -0.216339, 0.259761, -0.605796, -0.573465,
 0.180503, -0.0715446, 0.733475, -0.0134188, -0.427235, 0.0258062, 0.271239,
 -0.689291, 0.204698, 0.0870749, 0.126792, 0.233768, 0.0777638, -0.0526335,
 -0.414622, 0.678889, 0.135431, 0.127939, 0.674631, 0.179362, 0.314921, 0.342163,
 0.100506, -0.141703, 0.0753214, 0.394161, -0.959349, 0.141477, 0.0520813,
 -0.425997, 0.236845, -0.612993, 0.116102, 0.296879, 0.0653419, 0.00415566,
 -0.3011, 0.0807105, -0.283217, 0.634543, 0.260619, -0.466051, -0.146815,
 -0.216283, 0.0111701, 0.279145, 0.144997, -0.937292, -0.0328569, 0.0110048,
 0.0109061, 0.167212, 0.180531, 0.202897, -0.196003, 0.231155, -0.696144,
 0.121564, 0.175506, 0.195746, 0.614047, -0.140801, -0.404928, 0.427558,
 0.323712, 0.0847022, 0.0704572, 0.544923, -0.0229006, -0.507404, -0.310886,
 0.5274, -0.549036, -0.168166, -0.1132, -0.281934, -0.339257, 0.27623}
```

```
Clear[ListOfSampleStatistics];
ListOfSampleStatistics = SampleMeans;
Clear[xmin, xmax, a, b];
a = Min[ListOfSampleStatistics];
b = Max[ListOfSampleStatistics];
xmin = a - .1 * (b - a);
xmax = b + .1 * (b - a);
Clear[Model1];
Model1[x_] := PDF[NormalDistribution[0, 1 / Sqrt[nSize]], x];
(*   CLT model for samples from N[0,1]  *)
Print["Histogram of the sample means from our
   collection of samples, shown with absolute frequencies"]
Histogram[ListOfSampleStatistics]
Print["This collection of sample means is itself a sample, and "]
Print[
 "it should resemble the population from which the sample means are drawn, "]
Print["which is the sampling distribution of sample means."]
```

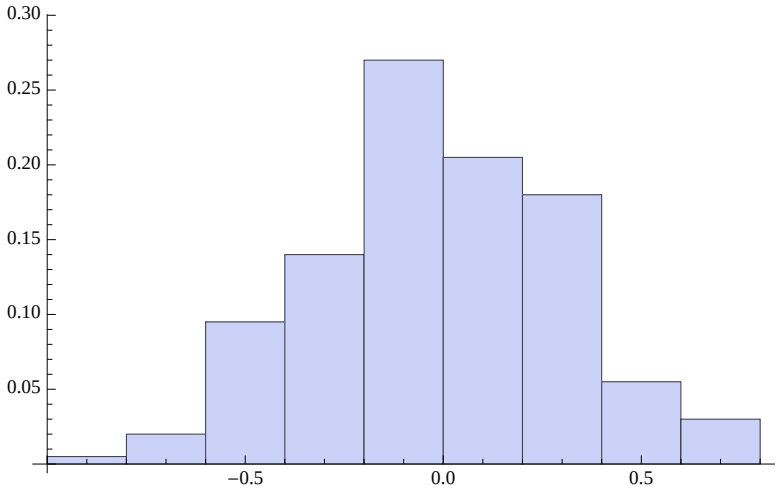
```
Histogram of the sample means from our
  collection of samples, shown with absolute frequencies
```




```
This collection of sample means is itself a sample, and
it should resemble the population from which the sample means are drawn,
which is the sampling distribution of sample means.
```

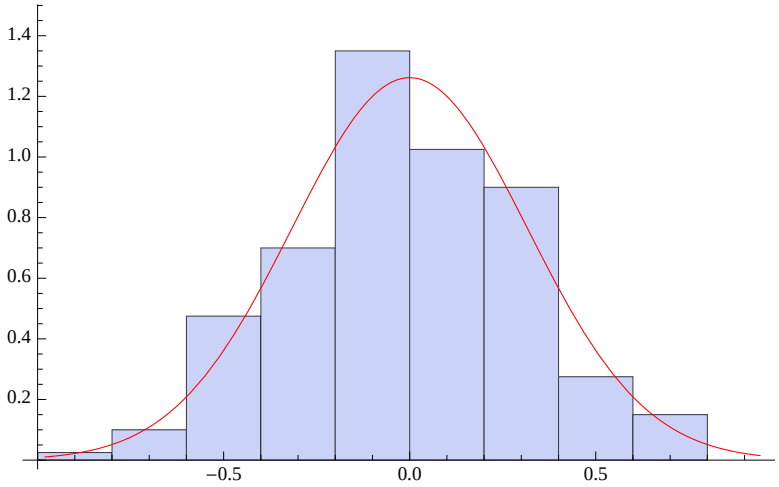
```
Print["Histogram of this collection
   of sample means, shown with relative frequencies"]
Histogram[ListOfSampleStatistics, Automatic, "Probability"]
```

Histogram of this collection of sample means, shown with relative frequencies




```
Print["Histogram of this collection of sample means, shown with
   heights of the absolute frequencies divided by the bin widths"]
Show[{Histogram[ListOfSampleStatistics, Automatic, "ProbabilityDensity"],
  Plot[Model1[x], {x, xmin, xmax}, PlotStyle → {Red}]     }]
Print["Red is CLT Model that applies to the population of sample means."]
```

Histogram of this collection of sample means, shown
with heights of the absolute frequencies divided by the bin widths

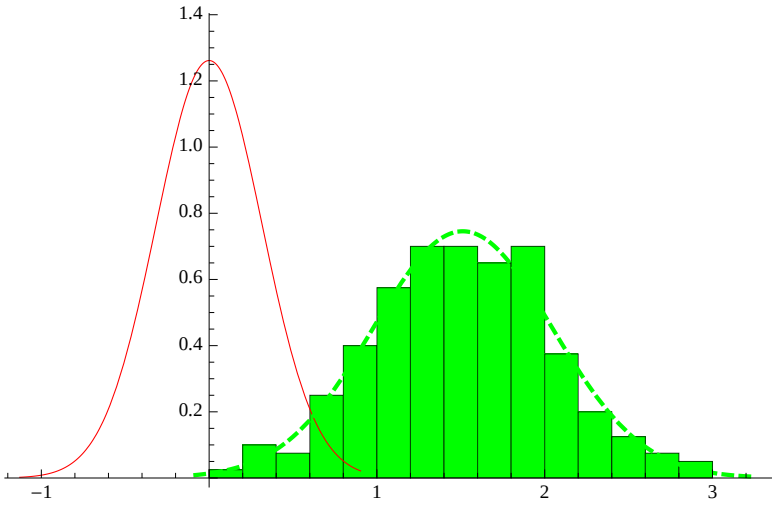


Red is CLT Model that applies to the population of sample means.

■ **sample maxima - NO Central Limit Theorem**

```
Clear[ListOfSampleStatistics];
ListOfSampleStatistics = SampleMaxs;
Clear[xmin3, xmax3];
a = Min[ListOfSampleStatistics];
b = Max[ListOfSampleStatistics];
xmin3 = a - .1 * (b - a);
xmax3 = b + .1 * (b - a);
Clear[Model3];
Model3[x_] := PDF[NormalDistribution[Mean[ListOfSampleStatistics],
   StandardDeviation[ListOfSampleStatistics]], x]
Print["Red is CLT Model.  Histogram and a normal
   comparison for the collection of sample maximums"]
Show[{Histogram[ListOfSampleStatistics, Automatic,
   "ProbabilityDensity", ChartStyle → Green],
  Plot[Model1[x], {x, xmin, xmax}, PlotStyle → {Red}],
  Plot[Model3[x], {x, xmin3, xmax3}, PlotStyle → {Green, Dashed, Thick}]
 }]
Print["The collection of sample maxs should resemble
   the population from which the sample maxs are drawn, "]
Print["which is the sampling distribution of sample means."]
```

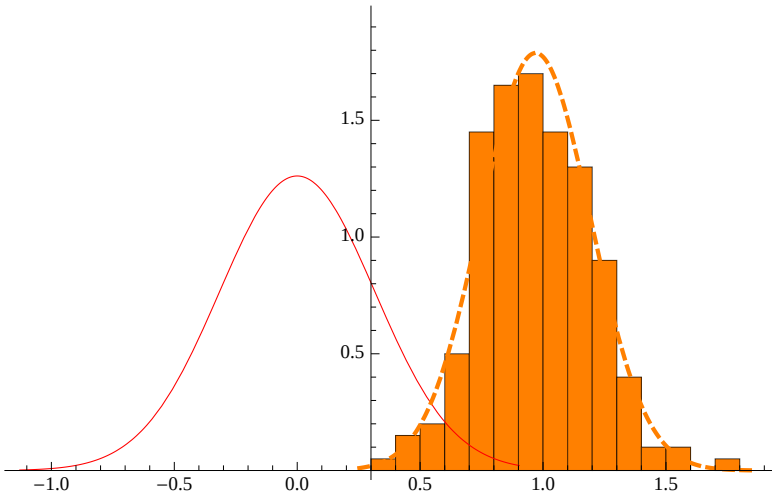
```
Red is CLT Model.  Histogram and a
  normal comparison for the collection of sample maximums
```



- **sample standard deviations - NO Central Limit Theorem**

```
Clear[ListOfSampleStatistics];
ListOfSampleStatistics = SampleStDevs;
Clear[xmin3, xmax3];
a = Min[ListOfSampleStatistics];
b = Max[ListOfSampleStatistics];
xmin3 = a - .1 * (b - a);
xmax3 = b + .1 * (b - a);
Clear[Model3];
Model3[x_] := PDF[NormalDistribution[Mean[ListOfSampleStatistics],
   StandardDeviation[ListOfSampleStatistics]], x]
Print["Red is CLT Model.  Histogram and a normal comparison
   for the collection of sample standard deviations"]
Show[{Histogram[ListOfSampleStatistics, Automatic,
   "ProbabilityDensity", ChartStyle → Orange],
  Plot[Model1[x], {x, xmin, xmax}, PlotStyle → {Red}],
  Plot[Model3[x], {x, xmin3, xmax3}, PlotStyle → {Orange, Dashed, Thick}]
 }]
```

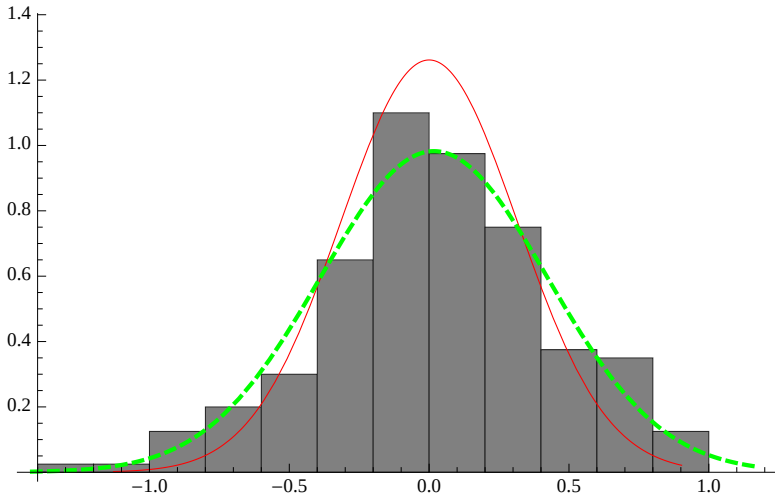
```
Red is CLT Model.  Histogram and a normal
  comparison for the collection of sample standard deviations
```



■ **sample medians - NO Central Limit Theorem**

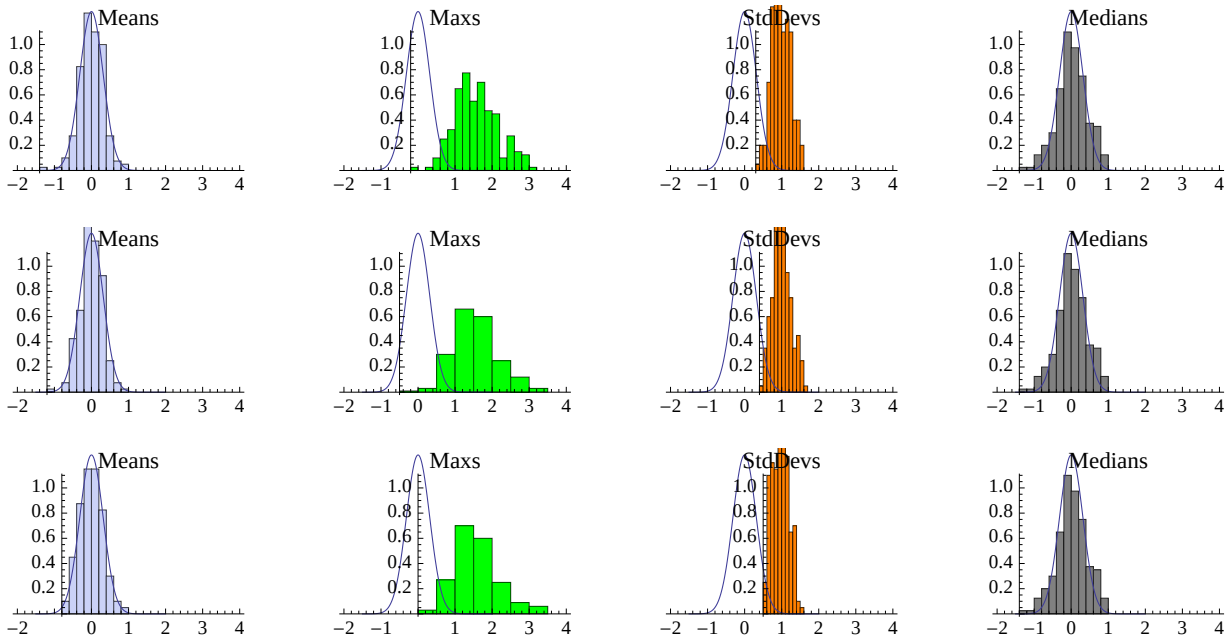
```
Clear[ListOfSampleStatistics];
ListOfSampleStatistics = SampleMedians;
Clear[xmin3, xmax3];
a = Min[ListOfSampleStatistics];
b = Max[ListOfSampleStatistics];
xmin3 = a - .1 * (b - a);
xmax3 = b + .1 * (b - a);
Clear[Model3];
Model3[x_] := PDF[NormalDistribution[Mean[ListOfSampleStatistics],
   StandardDeviation[ListOfSampleStatistics]], x]
Print["Red is CLT Model.  Histogram and a normal
   comparison for the collection of sample medians"]
Show[{Histogram[ListOfSampleStatistics, Automatic,
   "ProbabilityDensity", ChartStyle → Gray],
  Plot[Model1[x], {x, xmin, xmax}, PlotStyle → {Red}],
  Plot[Model3[x], {x, xmin3, xmax3}, PlotStyle → {Green, Dashed, Thick}]
 }]
```

```
Red is CLT Model.  Histogram and a
  normal comparison for the collection of sample medians
```



■ see the random variation in multiple data sets

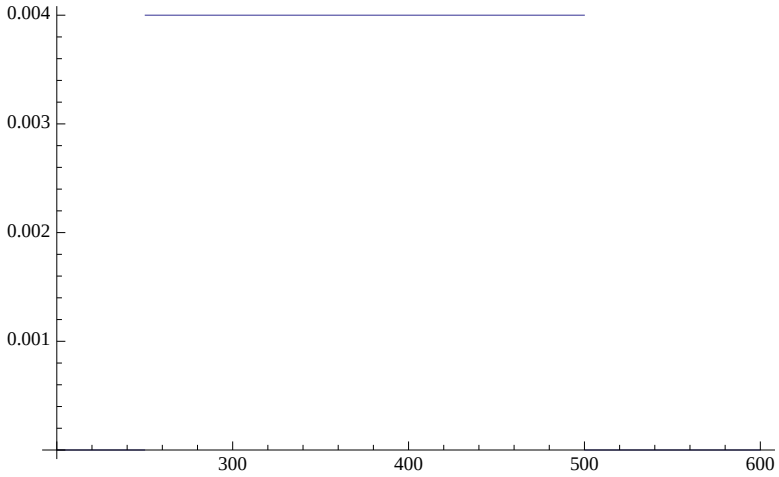
```
Do[
 Clear[SampleSets, SampleMeans, SampleStDevs, SampleMaxs, i];
 SampleMeans = {};
 SampleStDevs = {};
 SampleMaxs = {};
 For[i = 1, i ≤ numSets, i++,
  SampleSets[i] = RandomReal[NormalDistribution[0, 1], nSize];
  AppendTo[SampleMeans, Mean[SampleSets[i]] ];
  AppendTo[SampleStDevs, StandardDeviation[SampleSets[i]] ];
  AppendTo[SampleMaxs, Max[SampleSets[i]] ]  ];
 Print[GraphicsArray[{
    Show[{Histogram[SampleMeans, Automatic, "ProbabilityDensity"],
      Plot[Model1[x], {x, -1.5, 2}]},
     PlotRange → {{-2, 4}, {0, 1}}, PlotLabel → "Means"],
    Show[
     {Histogram[SampleMaxs, Automatic, "ProbabilityDensity", ChartStyle → Green],
      Plot[Model1[x], {x, -1.5, 2}]},
     PlotRange → {{-2, 4}, {0, 1}}, PlotLabel → "Maxs"],
    Show[{Histogram[SampleStDevs,
       Automatic, "ProbabilityDensity", ChartStyle → Orange],
      Plot[Model1[x], {x, -1.5, 2}]},
     PlotRange → {{-2, 4}, {0, 1}}, PlotLabel → "StdDevs"],
    Show[{Histogram[SampleMedians,
       Automatic, "ProbabilityDensity", ChartStyle → Gray],
      Plot[Model1[x], {x, -1.5, 2}]},
     PlotRange → {{-2, 4}, {0, 1}},
     PlotLabel → "Medians"]          }]        ],      {j, 1, 3}]
```



■ example 2 - sampling from a uniform distribution

```
Print["The parent population is a uniform distribution. "]
Plot[PDF[UniformDistribution[{250, 500}], x], {x, 200, 600}]
Clear[SampleSets2, SampleMeans2, SampleStDevs2, SampleMaxs2, nSize2, i, numSets2]
nSize2 = 10;
numSets2 = 200;
SampleMeans2 = {};
SampleStDevs2 = {};
SampleMaxs2 = {};
For[i = 1, i ≤ numSets2, i++,
 SampleSets2[i] = RandomReal[UniformDistribution[{250, 500}], nSize2];
 AppendTo[SampleMeans2, Mean[SampleSets2[i]] ];
 AppendTo[SampleStDevs2, StandardDeviation[SampleSets2[i]] ];
 AppendTo[SampleMaxs2, Max[SampleSets2[i]] ]
]
SampleSets2[3]
SampleMeans2[[3]]
SampleStDevs2[[3]]
SampleMaxs2[[3]]
```

```
The parent population is a uniform distribution.
```




```
{485.753, 259.802, 254.55, 355.981, 266.116, 430.22, 450.207, 497.068, 427.6, 256.43}

368.373

101.245

497.068
```

```
Clear[mu2, sigma2, sig2DivRootn];
mu2 = Mean[UniformDistribution[{250, 500}]]
sigma2 = Sqrt[ExpectedValue[(x - mu2)^2, UniformDistribution[{250, 500}], x] ]
```

```
375
```

$$\frac{125}{\sqrt{3}}$$

```
sig2DivRootn = sigma2 / Sqrt[nSize2]
```

$$25\sqrt{\frac{5}{6}}$$

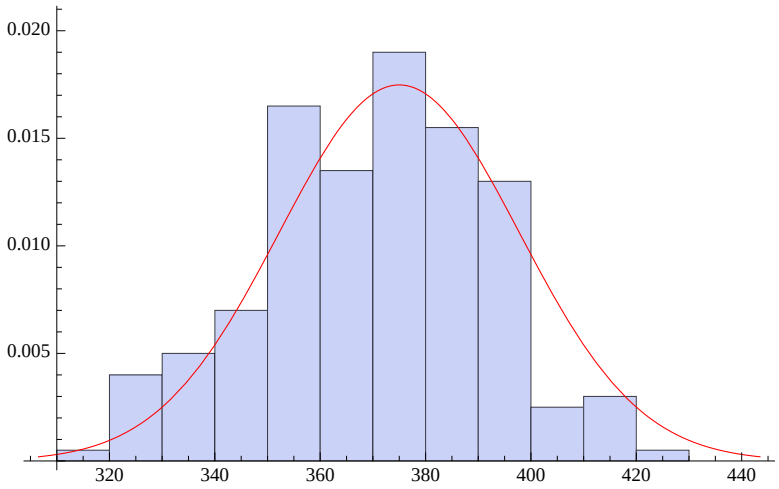
```
N[%]
```

```
22.8218
```

```
Clear[pxmin2, pxmax2];
pxmin2 = mu2 - 3 sig2DivRootn;
pxmax2 = mu2 + 3 sig2DivRootn;
```

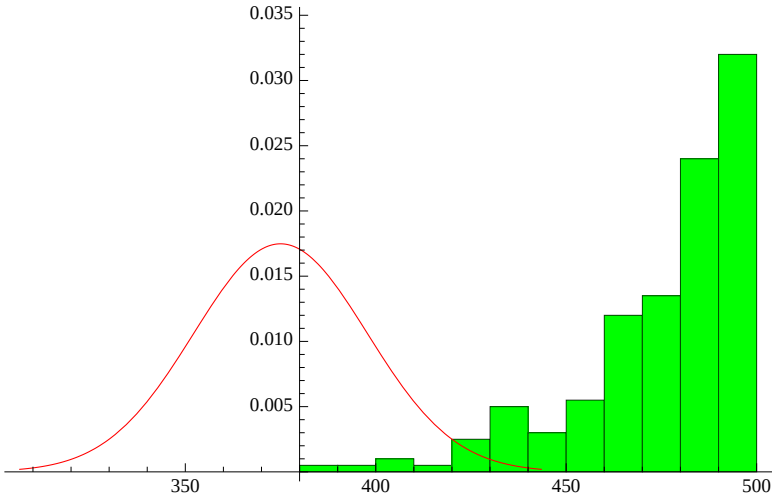
```
Clear[p2];
p2 = Plot[PDF[NormalDistribution[mu2, sig2DivRootn], x],
    {x, pxmin2, pxmax2}, PlotStyle → {Red}];
Print["Red is CLT Model.  Histogram for the collection of sample means"]
Show[{Histogram[SampleMeans2, Automatic, "ProbabilityDensity"],
  p2}]
```

```
Red is CLT Model.  Histogram for the collection of sample means
```



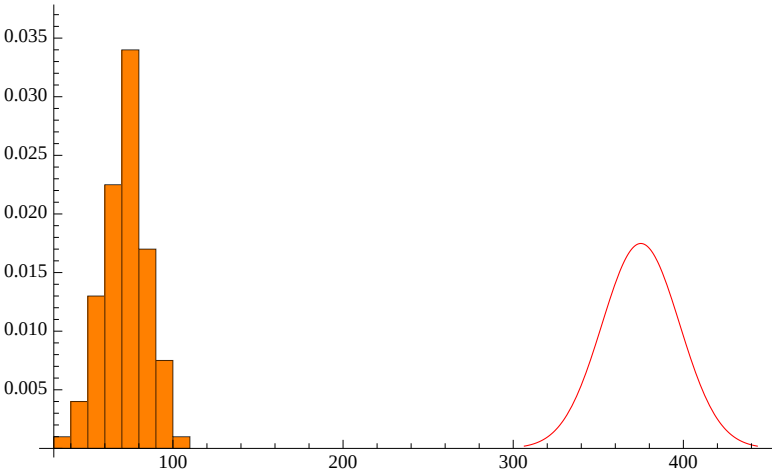
```
Print["Red is CLT Model.  Histogram for the collection of sample maximums"]
Show[{Histogram[SampleMaxs2, Automatic, "ProbabilityDensity", ChartStyle → Green],
  p2}]
```

```
Red is CLT Model.  Histogram for the collection of sample maximums
```




```
Print["Red is CLT Model.  Histogram
   for the collection of sample standard deviations"]
Show[{Histogram[SampleStDevs2, Automatic, "ProbabilityDensity",
   ChartStyle → Orange],
  p2}]
```

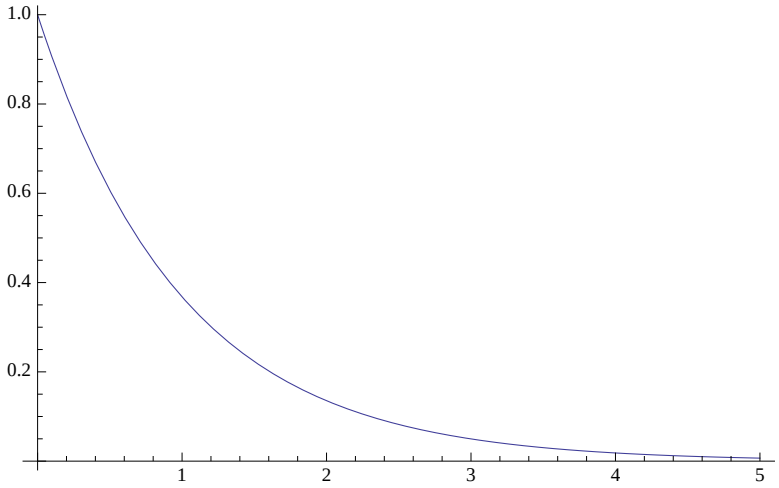
```
Red is CLT Model.  Histogram for the collection of sample standard deviations
```



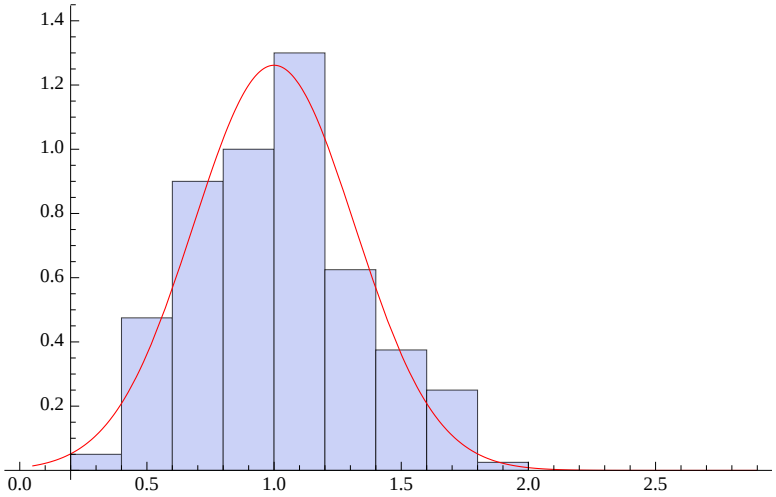
■ example 3 -- sampling from an exponential distribution

```
Print["The parent population is the exponential distribution. "]
Plot[PDF[ExponentialDistribution[1], x], {x, 0, 5}]
Clear[SampleSets3, SampleMeans3, SampleStDevs3, SampleMaxs3,
 nSize3, i, numSets3, mu3, sigma3, sig3DivRootn, pxmin3, pxmax3]
nSize3 = 10;
numSets3 = 200;
SampleMeans3 = {};
SampleStDevs3 = {};
SampleMaxs3 = {};
For[i = 1, i ≤ numSets3, i++,
 SampleSets3[i] = RandomReal[ExponentialDistribution[1], nSize2];
 AppendTo[SampleMeans3, Mean[SampleSets3[i]] ];
 AppendTo[SampleStDevs3, StandardDeviation[SampleSets3[i]] ];
 AppendTo[SampleMaxs3, Max[SampleSets3[i]] ]
]
mu3 = Mean[ExponentialDistribution[1]];
sigma3 = Sqrt[ExpectedValue[(x - mu3)^2, ExponentialDistribution[1], x] ];
sig3DivRootn = sigma3 / Sqrt[nSize3];
pxmin3 = mu3 - 3 sig3DivRootn;
pxmax3 = mu3 + 6 sig3DivRootn;
Clear[p3];
p3 = Plot[PDF[NormalDistribution[mu3, sig3DivRootn], x],
   {x, pxmin3, pxmax3}, PlotStyle → {Red}];
Print["Red is CLT Model.  Histogram for the collection of sample means"]
Show[{Histogram[SampleMeans3, Automatic, "ProbabilityDensity"], p3}]
```

The parent population is the exponential distribution.

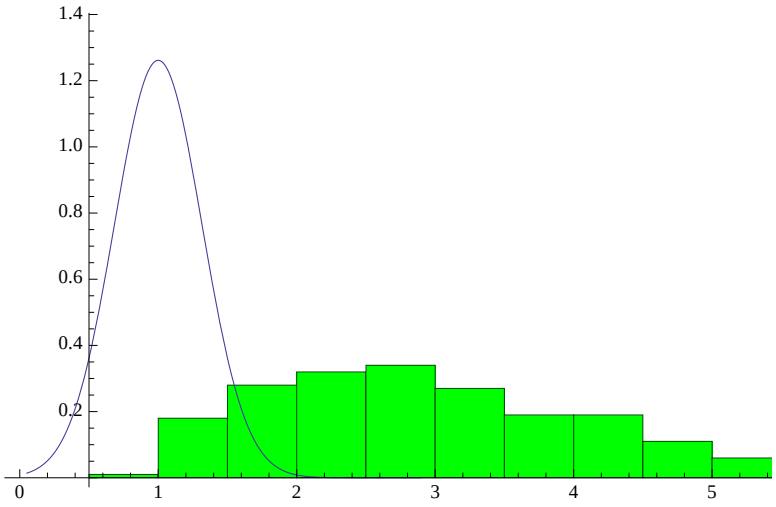


Red is CLT Model. Histogram for the collection of sample means




```
Print["Red is CLT Model.  Histogram for the collection of sample maximums"]
Show[{Histogram[SampleMaxs3, Automatic, "ProbabilityDensity", ChartStyle → Green],
  Plot[PDF[NormalDistribution[mu3, sig3DivRootn], x], {x, pxmin3, pxmax3}]}]
```

```
Red is CLT Model.  Histogram for the collection of sample maximums
```



```
Print["Red is CLT Model.  Histogram
   for the collection of sample standard deviations"]
Show[{Histogram[SampleStDevs3, Automatic, "ProbabilityDensity",
   ChartStyle → Orange],
  Plot[PDF[NormalDistribution[mu3, sig3DivRootn], x], {x, pxmin3, pxmax3}]}]
```

```
Red is CLT Model.  Histogram for the collection of sample standard deviations
```